International Journal of Engineering Research & Management Technology ISSN: 2348-4039

November-2021 Volume-8, Issue-6

Email: editor@ijermt.org www.ijermt.org

EOQ Model for Weibull Distributed Deteriorating Items with Learning Effect under Stock-Dependent Demand and Partial Backlogging.

Dr. Biswaranjan Mandal

Associate Professor of Mathematics

Acharya Jagadish Chandra Bose College, Kolkata, West Bengal, India

Abstract:

In the present paper, we deal with an economic order quantity (EOQ) inventory model for deteriorating items with stock dependent demand in which shortages are allowed and partially backlogged. The holding cost follows the learning curve. Here we have considered two parameter Weibull (Swedish engineer Wallodi Weibull) distribution deterioration. The stock-dependent demand pattern is considered in the present inventory system. The model is maximized to the total average profit by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters has been presented.

Key Words: EOQ, learning effect, Weibull, deteriorating, stock-dependent, shortages and partial backlogging.

Subject classification: AMS Classification No. 90B05

1. Introduction:

Almost all items deteriorate with time excepting items like steel, hardware, glassware etc. The rate of deterioration is very small that there is hardly any need to assume the effect of deterioration. In many inventory models, this effect is most important. Ghare & Schrader [1] developed an inventory model assuming a constant rate of deterioration. This assumption was relaxed by Covert & Philip [2] assuming a two parameter Weibull distribution to represent the distribution of the time of deterioration. Later Mishra [3], A. K.Jalan et al [4], Dr. Biswaranjan Mandal [5], P. R. Tadikamalla [6] etc are mentioned a few.

It is very well known to the fact that the customers in a supermarket have been influenced by the stock level. The demand may vary if the stock level increases or decreases over time and so the stock-dependent demand plays an important role in the corporate world. The reason behind this phenomenon is a typical psychology of customers. This may occur due to the fact of feeling of obtaining a wide range for selection when a large amount is stored or displayed or having doubt about the freshness or quality of the product when a small amount is stored. Based on the observed phenomenon, it is cleared that in real life the demand rate of items may be influenced by the stock levels. Many researchers like J. T. Teng et al [7], T. K. Datta et al[8], B. R. Singh [9], R. P. Tripathi et al [10], N. Sharma et al [11] etc have considered stock dependent demand rate in their models.

In practice, some customers would like to wait for backlogging during the shortage period, but others would not. Consequently, the opportunity cost due to lost sales should be considered in the inventory model. Again in some inventory systems for few fashionable commodities, the length of waiting time for the next replenishment would determine whether backlogging will be accepted or not. So backlogging rate should be variable and dependent on the length of the waiting time for the next replenishment. Abad [12] developed an EOQ model assuming shortages which are partially backlogged. Late many researchers like H J Chang et al [13], K. S. Wu et al [14], Dr. Biswaranjan Mandal [15] etc developed inventory policies by considering "time proportional partial backlogging rate".

The effect of learning has a direct impact on profit or loss, and it is a promotional deemed effective tool for inventory management. Few research areas of N. Kumar [16], M. K. Sharma et al [17] etc are important in the field of inventory management.

In view of the above sort of situations and facts, the present paper deals with an inventory model for Weibull deteriorating items with stock dependent demand in which shortages are allowed and partially backlogged. The holding cost follows the learning curve. The model is maximized to the total average profit by finding optimal values and finally it is illustrated by a numerical example along with the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters.

2. Assumptions and Nomenclatures:

2.1 Assumptions:

The present inventory model is developed under the following assumptions:

i. Lead time is zero.
ii. Replenishment rate is infinite but size is finite.
iii. The time horizon is finite.
iv. There is no repair of deteriorated items occurring during the cycle.
v. Deterioration occur when the item is effectively in stock.
vi. The demand rate for the product is assumed as stock dependent.
vii. Holding cost follows the learning curve.
viii. Shortages are allowed and partial backlogged.

2.2 Nomenclatures:

The following nomenclatures are used in the proposed model:

i. q(t) : On hand inventory at time t.
ii. D(t) : Demand rate $D(t)=\left|\begin{matrix} a+bq(t), q(t)>0 \\ a, q(t)\leq 0 \end{matrix}\right\rangle$, where a(>0) be the initial demand and b(0<b<<1) be a constant parameter.
iii. B(t) : Backlogging rate $B(t)=e^{-\delta(T-t)}$, where $\delta(>0)$ is a constant.
ii. Q : On-hand inventory.
iii. $\theta(t)=\alpha\beta t^{\beta-1}, t\geq 0$ is the two parameter Weibull distribution deterioration rate function where $\alpha(0<\alpha<<1)$ is a scale parameter and $\beta(>0)$ is a shape parameter.
iv. t_d : The time length in which the product has no deterioration (fresh product time).
v. t_1 : The time length in which the stock is completely diminished.
vi. T : The fixed length of each production cycle.

vii. A_0 : The ordering cost per order during the cycle period.

viii. p_c : The purchasing cost per unit item.

ix. $h_0 + \frac{h_1}{n^{\alpha_2}}$: The holding cost with learning effect.

x. d_c : The deterioration cost per unit item.

xi. c_s : The shortage cost per unit item.

xii. o_c : The opportunity cost per unit item.

xiii. r_c : Sales revenue cost per unit time.

xiv. $q(t) = \left| \begin{matrix} q_1(t), 0 \le t \le t_d \\ q_2(t), t_d \le t \le t_1 \\ q_3(t), t_1 \le t \le T \end{matrix} \right\rangle$

xv. TP : Average total profit per unit time.

3. Mathematical Formulation and Solution:

In this model, the cycle starts attaining maximum inventory level with no shortage. The total time is distributed into three time intervals. During $[0, t_d]$, the inventory level is depleted only due to stock -dependent demand rate. From t = t_d to t = t_1, the stock will be diminished due to the effect of amelioration, deterioration and demand, and ultimately falls to zero at t = t_1. The shortages occur during time period [t_1, T] which are partially backlogged. The differential equations pertaining to the above situations are given by

$$\frac{dq_1(t)}{dt} = -[a + bq_1(t)], 0 \le t \le t_d \tag{3.1}$$

$$\frac{dq_2(t)}{dt} + \alpha\beta t^{\beta-1} q_2(t) = -[a + bq_2(t)], t_d \le t \le t_1 \tag{3.2}$$

And $$\frac{dq_3(t)}{dt} = -ae^{-\delta(T-t)}, t_1 \le t \le T \tag{3.3}$$

The initial conditions are $q_1(0) = Q_1$, $q_2(t_1) = 0$, $q_3(t_1) = 0$, $q_3(T) = -Q_2$ and $Q = Q_1 + Q_2$ (3.4)

Since $\alpha(0 < \alpha << 1)$ and b(0<b<<1) , we ignore the terms $O(\alpha^2), O(b^2)$ and $O(\alpha b)$, then the solutions of the equations (3.1), (3.2) and (3.3) using (3.4) are given by the following

$$q_1(t) = Q_1(1 - bt) - at, \qquad 0 \le t \le t_d \tag{3.5}$$

$$q_2(t) = a(t_1 - t) + \frac{ab}{2}(t_1^2 - 2t_1 t + t^2) + \frac{a\alpha}{\beta+1}\{t_1^{\beta+1} - (\beta+1)t_1 t^\beta + \beta t^{\beta+1}\},$$
$$t_d \le t \le t_1 \tag{3.6}$$

And $$q_3(t) = -a\{t - \delta(Tt - \frac{t^2}{2})\} + a\{t_1 - \delta(Tt_1 - \frac{t_1^2}{2})\}, \ t_1 \le t \le T \tag{3.7}$$

Since $q_1(t_d) = q_2(t_d)$, we get the following expression of on-hand inventory from the equations (3.5) and (3.6) neglecting second and higher order powers of α and b,

$$Q_1 = at_d + a(1+bt_d)(t_1 - t_d) + \frac{ab}{2}(t_1^2 - 2t_1t_d + t_d{}^2) + \frac{a\alpha}{\beta+1}\{t_1^{\beta+1} - (\beta+1)t_1t_d{}^\beta + \beta t_d{}^{\beta+1}\}, \quad (3.8)$$

Again $Q_2 = -I_3(T)$ gives the following expression using (3.7)

$$Q_2 = a(T - \delta\frac{T^2}{2}) - a\{t_1 - \delta(Tt_1 - \frac{t_1{}^2}{2})\} \quad (3.9)$$

Therefore the total on-hand inventory is given by

$$Q = Q_1 + Q_2$$

Or, Q = $at_d + a(1+bt_d)(t_1 - t_d) + \frac{ab}{2}(t_1^2 - 2t_1t_d + t_d{}^2) + \frac{a\alpha}{\beta+1}\{t_1^{\beta+1} - (\beta+1)t_1t_d{}^\beta + \beta t_d{}^{\beta+1}\}$

$$+ a(T - \delta\frac{T^2}{2}) - a\{t_1 - \delta(Tt_1 - \frac{t_1{}^2}{2})\} \quad (3.10)$$

4. Cost Components:

The total profit over the period [0, T] consists of the following cost components:

(1). Ordering cost **(OC)** over the period [0,T] = A_0 (fixed) (4.1)

(2). Purchasing cost **(PC)** over the period [0,T] = p_c Q

Or, PC = $p_c[at_d + a(1+bt_d)(t_1 - t_d) + \frac{ab}{2}(t_1{}^2 - 2t_1t_d + t_d{}^2) + \frac{a\alpha}{\beta+1}\{t_1{}^{\beta+1} - (\beta+1)t_1t_d{}^\beta + \beta t_d{}^{\beta+1}\}$

$$+ a(T - \delta\frac{T^2}{2}) - a\{t_1 - \delta(Tt_1 - \frac{t_1{}^2}{2})\}\ ] \quad (4.2)$$

(3). Holding cost for carrying inventory **(HC)** over the period [0,T]

$$\text{HC} = (h_0 + \frac{h_1}{n^{\alpha_2}})\,[\int_0^{t_d} q_1(t)dt + \int_{t_d}^{t_1} q_2(t)dt\ ]$$

Putting the values of $q_1(t)$ and $q_2(t)$ from (3.5) and (3.6), and integrating and then substituting the value of Q_1 from (3.8), we get the following expression after neglecting second and higher order powers of α and b

$$\text{HC} = (h_0 + \frac{h_1}{n^{\alpha_2}})\ \Big[\frac{a}{2}t_d^2(1 - bt_1) + at_d(1+bt_d)(t_1 - t_d) + \frac{ab}{2}t_d(t_1 - t_d)^2 + \frac{a\alpha}{\beta+1}t_d\{t_1^{\beta+1} - (\beta+1)t_1t_d^\beta + \beta t_d^{\beta+1}\}$$

$$+ \frac{a}{2}(t_1 - t_d)^2 + \frac{ab}{6}(t_1 - t_d)^3 + \frac{a\alpha}{(\beta+1)(\beta+2)}\{\beta t_1{}^{\beta+2} - (\beta+2)t_1^{\beta+1}t_d + (\beta+2)t_1t_d{}^{\beta+1} - \beta t_d{}^{\beta+2}\}\Big] \quad (4.3)$$

(4). Cost due to deterioration **(CD)** over the period [0,T]

$$\text{CD} = d_c\int_{t_d}^{t_1} \alpha\beta t^{\beta-1} q_2(t)dt$$

Putting the value $q_2(t)$ from (3.6), and then integrating and neglecting second and higher order powers of α and b we get the following

$$CD = d_c a\alpha[\frac{1}{\beta+1}t_1^{\beta+1} - t_1 t_d^{\beta} + \frac{\beta}{\beta+1}t_d^{\beta+1}] \qquad (4.4)$$

(5). Cost due to shortage **(CS)** over the period [0,T]

$$CS = -c_s \int_{t_1}^{T} q_3(t)dt$$

Putting the value $q_3(t)$ from (3.7), and then integrating we get the following

$$CS = c_s a[\frac{\delta}{3}t_1^3 - \frac{\delta}{3}T^3 + \frac{1}{2}t_1^2 + \frac{1}{2}T^2 - Tt_1 + \delta T^2 t_1 - \delta T {t_1}^2] \qquad (4.5)$$

(6). Opportunity Cost due to lost sales **(OPC)** over the period [0,T]

$$OPC = o_c \int_{t_1}^{T} a(1-e^{-\delta(T-t)})dt$$

$$= o_c a[T - t_1 - \frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\}] \qquad (4.6)$$

(7). Sales Revenue Cost **(SRC)** over the period [0,T]

$$SRC = r_c[\int_0^{t_d}\{a+bq_1(t)\}dt + \int_{t_d}^{t_1}\{a+bq_2(t)\}dt + \int_{t_1}^{T} ae^{-\delta(T-t)}dt]$$

Putting the values of $q_1(t)$ and $q_2(t)$ from (3.5) and (3.6), and integrating and then substituting the value of Q_1 from (3.8), we get the following

$$SRC = r_c a \Big[t_1 + \frac{b}{2}t_d^2(1-bt_1) + bt_d(1+bt_d)(t_1-t_d) + \frac{b^2}{2}t_d(t_1-t_d)^2 + \frac{b\alpha}{\beta+1}t_d\{{t_1}^{\beta+1} - (\beta+1)t_1{t_d}^{\beta} + \beta {t_d}^{\beta+1}\}$$
$$+ \frac{1}{2}(t_1-t_d)^2 + \frac{b^2}{6}(t_1-t_d)^3 + \frac{b\alpha}{(\beta+1)(\beta+2)}\{\beta {t_1}^{\beta+2} - (\beta+2){t_1}^{\beta+1}t_d + (\beta+2)t_1{t_d}^{\beta+1} - \beta {t_d}^{\beta+2}\}$$
$$+ \frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\} \Big] \qquad (4.7)$$

Thus the average total profit per unit time of the system during the cycle [0,T] will be

$$TP(t_1) = \frac{1}{T} \text{ [SRC-OC –PC-HC-CD-CS-OPC]}$$

$$= \frac{1}{T}\Big[r_c a\,[t_1 + \frac{b}{2}t_d^2(1-bt_1) + bt_d(1+bt_d)(t_1-t_d) + \frac{b^2}{2}t_d(t_1-t_d)^2 + \frac{b\alpha}{\beta+1}t_d\{{t_1}^{\beta+1} - (\beta+1)t_1{t_d}^{\beta} + \beta {t_d}^{\beta+1}\}$$
$$+ \frac{1}{2}(t_1-t_d)^2 + \frac{b^2}{6}(t_1-t_d)^3 + \frac{b\alpha}{(\beta+1)(\beta+2)}\{\beta {t_1}^{\beta+2} - (\beta+2){t_1}^{\beta+1}t_d + (\beta+2)t_1{t_d}^{\beta+1} - \beta {t_d}^{\beta+2}\}$$
$$+ \frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\}] - A_0 - p_c[at_d + a(1+bt_d)(t_1-t_d) + \frac{ab}{2}(t_1^2 - 2t_1t_d + {t_d}^2)$$

$$+\frac{a\alpha}{\beta+1}\{t_1^{\beta+1}-(\beta+1)t_1t_d^{\beta}+\beta t_d^{\beta+1}\} + a(T-\delta\frac{T^2}{2})-a\{t_1-\delta(Tt_1-\frac{t_1^2}{2})\}\ ] - (h_0+\frac{h_1}{n^{\alpha_2}})\ [$$

$$\frac{a}{2}t_d^2(1-bt_1)+at_d(1+bt_d)(t_1-t_d)+\frac{ab}{2}t_d(t_1-t_d)^2+\frac{a\alpha}{\beta+1}t_d\{t_1^{\beta+1}-(\beta+1)t_1t_d^{\beta}+\beta t_d^{\beta+1}\}$$

$$+\frac{a}{2}(t_1-t_d)^2+\frac{ab}{6}(t_1-t_d)^3+\frac{a\alpha}{(\beta+1)(\beta+2)}\{\beta t_1^{\beta+2}-(\beta+2)t_1^{\beta+1}t_d+(\beta+2)t_1t_d^{\beta+1}-\beta t_d^{\beta+2}\}\ ]$$

$$-d_c a\alpha[\frac{1}{\beta+1}t_1^{\beta+1}-t_1t_d^{\beta}+\frac{\beta}{\beta+1}t_d^{\beta+1}]-c_s a[\frac{\delta}{3}t_1^3-\frac{\delta}{3}T^3+\frac{1}{2}t_1^2+\frac{1}{2}T^2-Tt_1+\delta T^2t_1-\delta Tt_1^2]$$

$$-\quad -o_c a[T-t_1-\frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\}]\ \Big] \qquad (4.8)$$

To maximize the profit, the necessary condition is $\frac{dTP(t_1)}{dt_1}=0$

This gives

$$\{r_c-p_c\delta+o_c+c_sT(1-\delta T)\}+(r_cb-h_0-\frac{h_1}{n^{\alpha_1}})[\{t_d-\frac{b}{2}t_d^2+bt_1t_d+\alpha t_d(t_1^{\beta}-t_d^{\beta})\}/2$$

$$+(t_1-t_d)+\frac{b}{2}(t_1-t_d)^2+\frac{\alpha}{\beta+1}(\beta t_1^{\beta+1}-(\beta+1)t_1^{\beta}t_d+t_d^{\beta+1})]\ -\{(b-\delta)p_c+c_s-2c_s\delta T\}t_1-c_s\delta t_1^2$$

$$-(r_c+o_c)e^{-\delta(T-t_1)}-\alpha(p_c+d_c)(t_1^{\beta}-t_d^{\beta})=0 \qquad (4.9)$$

For maximum, the sufficient condition $\frac{d^2TP(t_1)}{dt_1^2}$ <0 would be satisfied.

Let $t_1=t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q and TP^* of TP are obtained from the expressions (3.10) and (4.8) by putting the value $t_1=t_1^*$.

5. Some Special Cases:

(a). Absence of deterioration :

If the deterioration of items is switched off i.e. $\alpha=0$, then the expressions (3.10) and (4.8) of on-hand inventory(Q) and average total profit per unit time (TP(t_1)) during the period [0,T] become

$$Q=\ at_d+a(1+bt_d)(t_1-t_d)+\frac{ab}{2}(t_1^2-2t_1t_d+t_d^2)+a(T-\delta\frac{T^2}{2})-a\{t_1-\delta(Tt_1-\frac{t_1^2}{2})\} \qquad (5.1)$$

And
$$TP(t_1)=\frac{1}{T}\Big[\ r_ca\ [\ t_1+\frac{b}{2}t_d^2(1-bt_1)+bt_d(1+bt_d)(t_1-t_d)+\frac{b^2}{2}t_d(t_1-t_d)^2+\frac{1}{2}(t_1-t_d)^2+\frac{b^2}{6}(t_1-t_d)^3+$$

$$\frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\}\ ]\ -\ A_0\ -\ p_c[at_d+a(1+bt_d)(t_1-t_d)+\frac{ab}{2}(t_1^2-2t_1t_d+t_d^2)+a(T-\delta\frac{T^2}{2})-a\{t_1-\delta(Tt_1-\frac{t_1^2}{2})\}\ ]\ -$$

$$(h_0+\frac{h_1}{n^{\alpha_2}})\ [\ \frac{a}{2}t_d^2(1-bt_1)+at_d(1+bt_d)(t_1-t_d)+\frac{ab}{2}t_d(t_1-t_d)^2+\frac{a}{2}(t_1-t_d)^2+\frac{ab}{6}(t_1-t_d)^3\ ]$$

$$-o_ca[T-t_1-\frac{1}{\delta}\{1-e^{-\delta(T-t_1)}\}]\ \Big] \qquad (5.2)$$

The equation (4.9) becomes

$$\{r_c - p_c\delta + o_c + c_sT(1-\delta T)\} + (r_c b - h_0 - \frac{h_1}{n^{\alpha_1}})\frac{1}{2}[t_d - \frac{b}{2}t_d^2 + bt_1t_d + (t_1 - t_d) + \frac{b}{2}(t_1 - t_d)^2] - \{(b-\delta)p_c + c_s - 2c_s\delta T\}t_1 - c_s\delta t_1^2 - (r_c + o_c)e^{-\delta(T-t_1)} = 0 \quad (5.3)$$

This gives the optimum value of t_1.

(b). Constant demand rate:

If the demand rate is constant in nature i.e. b=0, then the expressions (3.10) and (4.8) of on-hand inventory (Q) and average total profit per unit time (TP(t_1)) during the period [0,T] become

$$Q = at_d + a(t_1 - t_d) + \frac{a\alpha}{\beta+1}\{t_1^{\beta+1} - (\beta+1)t_1t_d^{\beta} + \beta t_d^{\beta+1}\} + a(T - \delta\frac{T^2}{2}) - a\{t_1 - \delta(Tt_1 - \frac{t_1^2}{2})\} \quad (5.4)$$

And

$$TP(t_1) = \frac{1}{T}\Big[r_c a[t_1 + \frac{1}{2}(t_1 - t_d)^2 + \frac{1}{\delta}\{1 - e^{-\delta(T-t_1)}\}] - A_0 - p_c[at_d + a(t_1 - t_d) + \frac{a\alpha}{\beta+1}\{t_1^{\beta+1} - (\beta+1)t_1t_d^{\beta} + \beta t_d^{\beta+1}\} + a(T - \delta\frac{T^2}{2}) - a\{t_1 - \delta(Tt_1 - \frac{t_1^2}{2})\}]$$

$$- (h_0 + \frac{h_1}{n^{\alpha_2}})[\frac{a}{2}t_d^2 + at_d(t_1 - t_d) + \frac{a\alpha}{\beta+1}t_d\{t_1^{\beta+1} - (\beta+1)t_1t_d^{\beta} + \beta t_d^{\beta+1}\} + \frac{a}{2}(t_1 - t_d)^2 + \frac{a\alpha}{(\beta+1)(\beta+2)}\{\beta t_1^{\beta+2} - (\beta+2)t_1^{\beta+1}t_d + (\beta+2)t_1t_d^{\beta+1} - \beta t_d^{\beta+2}\}]$$

$$- d_c a\alpha[\frac{1}{\beta+1}t_1^{\beta+1} - t_1t_d^{\beta} + \frac{\beta}{\beta+1}t_d^{\beta+1}] - c_s a[\frac{\delta}{3}t_1^3 - \frac{\delta}{3}T^3 + \frac{1}{2}t_1^2 + \frac{1}{2}T^2 - Tt_1 + \delta T^2t_1 - \delta Tt_1^2]$$

$$- o_c a[T - t_1 - \frac{1}{\delta}\{1 - e^{-\delta(T-t_1)}\}]\Big] \quad (5.5)$$

The equation (4.9) becomes

$$\{r_c - p_c\delta + o_c + c_sT(1-\delta T)\} - (h_0 + \frac{h_1}{n^{\alpha_1}})\frac{1}{2}[t_d + \alpha t_d(t_1^{\beta} - t_d^{\beta}) + (t_1 - t_d) + \frac{\alpha}{\beta+1}(\beta t_1^{\beta+1} - (\beta+1)t_1^{\beta}t_d + t_d^{\beta+1})] - (-\delta p_c + c_s - 2c_s\delta T)t_1 - c_s\delta t_1^2 - (r_c + o_c)e^{-\delta(T-t_1)} - \alpha(p_c + d_c)(t_1^{\beta} - t_d^{\beta}) = 0 \quad (5.6)$$

This gives the optimum value of t_1.

6. Numerical Analysis:

To exemplify the above model numerically, let the values of parameters be as follows:

a = 200; b = 0.02; α = 0.01; β = 2; δ = 10; A_0 = \$ 300 per order; p_c = \$ 5 per unit, h_0 = \$ 2.5 per unit; h_1 = \$ 2.5 per unit; n = 3; α_2 = 0.1; d_c = \$9 per unit; c_s = \$10 per unit; o_c = \$12 per unit; r_c = \$ 12 per unit; t_d = 0.2 year and T = 1 year

Solving the equation (4.9) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.71 year; Q^* = 118.75 units and TP^* = Rs. 424.73

It is checked that this solution satisfies the sufficient condition for optimality.

7. Sensitivity Analysis and Discussion:

We now study the effects of changes in the system parameters a, b, α, β, δ, p_c, h_0, h_1, n, α_2, d_c, c_s, o_c and r_c on the optimal ordering quantity (Q^*) and the optimal total profit in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in	
		Q^*	TP^*
a	-50 -20 +20 +50	-50 -20 20 50	-85.32 -34.13 34.13 85.53
b	-50 -20 +20 +50	-0.21 -0.08 0.08 0.21	-0.78 -0.32 0.32 0.79
α	-50 -20 +20 +50	-0.30 -0.08 0.13 0.31	0.37 0.15 -0.15 -0.37
β	-50 -20 +20 +50	0.10 0.07 -0.07 -0.17	-0.24 -0.08 0.12 0.28
δ	-50 -20 +20 +50	-37.62 -10.79 8.01 16.88	-39.40 -10.04 6.89 24.01
p_c	-50 -20 +20 +50	-59.59 -19.96 15.98 34.25	9.98 3.72 -0.96 -9.72
h_0	-50 -20 +20 +50	-4.67 -1.89 1.93 4.88	15.02 6.07 -6.15 -15.51
h_1	-50 -20 +20 +50	-4.19 -1.70 1.69 4.32	13.48 5.42 -5.50 -13.87
n	-50 -20 +20 +50	-0.45 -0.29 0.34 0.63	-1.92 -0.62 0.63 1.05
α_2	-50 -20	0.51 0.19	-1.55 - 0.61

	+20 +50	-0.19 -0.45	0.60 1.46
d_c	-50 -20 +20 +50	-0.13 -0.05 0.04 0.13	0.21 0.08 -0.06 -0.21
c_s	-50 -20 +20 +50	-23.41 -7.48 19.16 21.37	-10.72 -3.60 4.00 6.92
o_c	-50 -20 +20 +50	34.56 13.89 -9.33 -35.39	42.41 19.56 -22.68 -62.10
r_c	-50 -20 +20 +50	35.12 14.09 -14.21 -35.80	-239.84 - 94.10 90.88 221.63

Analyzing the results of table A, the following observations may be made:

(i) The optimum average profit TP^* increase or decrease with the increase or decrease in the values of the system parameters a , b , β , δ , n , α_2 , c_s , o_c and r_c . On the other hand TP^* increase or decrease with the decrease or increase in the values of the system parameters α , p_c , h_0 , h_1 and d_c . The results obtained show that TP^* is very highly sensitive to changes in the value of parameters a, δ , h_0 , h_1 , o_c and r_c ; moderate sensitive towards changes of parameters p_c and c_s ; and less sensitive to the changes of parameters b, α , β , n, α_2 and d_c .

(ii) The optimum ordering quantity Q^* increase or decrease with the increase or decrease in the values of the system parameters a, b, α , δ , p_c , h_0 , h_1 , n, d_c and c_s . On the other hand Q^* increase or decrease with the decrease or increase in the values of the system parameters β , α_2 , o_c and r_c . The results obtained show that Q^* is very highly sensitive to changes in the value of parameters a, δ , p_c , c_s , o_c and r_c ; moderate sensitive towards changes of parameters h_0 and h_1 ; and less sensitive to the changes of parameters b, α , β , n, α_2 and d_c .

From the above analysis, it is seen that a, δ , o_c and r_c are very sensitive parameters in the sense that any error in the estimation of these parameters result in significant errors in the optimal solution. Hence estimation of the parameters a, δ , o_c and r_c need adequate attention .

References:

[1]. P. M Ghare and G. F. Schrader, “A model for exponentially decaying inventories”, J. Ind. Eng, 14, 1963, pp. 238-243.

[2] R P Covert and G C Philip, “An EOQ model for items with Weibull distribution deterioration”, AIIE Trans., 5, 1973, pp. 323-326.

[3] R. B. Mishra, “Optimum production lot-size model for a system with deteriorating inventory”, Int. J. Prod. Res., 13, 1975, pp. 495-505.

[4] A. K. Jalan, R R Giri and K S Chaudhuri, "EOQ model for item with Weibull distribution deterioration, shortages and trended demand", Int. J. Syst. Sci., 27(9), 1996, pp. 851-855.

[5] Dr. Biswaranjan Mandal, "Optimal inventory management for items with Weibull distributed deterioration under alternating demand rates", Int. J. Engng. Tech Res., 2(4), 2014, pp. 359-364.

[6] P R. Tadikamalla, "Applications of the Weibull Distribution in Inventory Control", https://doi.org/10.1057/jors.1978.11, 2017, pp 77-83.

[7] J. T. Teng and C. T. Chang, " EPQ model for deteriorating items with price and stock-dependent demand" Computers and Operations Research, 31, 1995, pp. 297-308.

[8] T. K. Datta and A. K. Pal, "Demand proportion by up-gradation under stock-dependent demand situation- a model", Int. J Prod. Eco. , 55, 1998, pp. 31-38.

[9] B. R. Singh, C. Singh and T. J. Singh, "Optimal policy for decaying items with stock-dependent demand under inflation in a supply chain", Int. review of Pure and Applied Mathematics, 3(2), 2007, pp. 189-197.

[10] R. P. Tripathi , D. SINGH and A Surbhi, "Inventory models for stock-dependent demand and time varying holding cost under different trade credits", Yugoslav Journal of Operations Research, DOI: https://doi.org/10.2298/YJOR160317018T, 2017, pp. 1-13.

[11] N. Sharma and P. Vrat, "Inventory models with stock-dependent demand: a comprehensive review and its linkage with waste management", International Journal of Inventory Research, 5(2), 2018, pp.112 – 152

[12] P. L. Abad, " Optimal pricing and lot sizing under conditions of perishability and partial backlogging", Mgmt Sci, 42(8), 1996, pp. 1093-1104.

[13] H J Chang and C Y Dye, " An EOQ model for deteriorating items with time-varying demand and partial backlogging", J Oper. Res. Soc., 50(11), 1999, pp. 1176-1182.

[14] K S Wu, L Y Ouyang and C T Yang, " An optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial back logging", Int. J. Prod. Eco, 101, 2006, pp. 369-384.

[15] Dr Biswaranjan Mandal, "An EOQ Model for Deteriorating and Ameliorating Items under Exponentially Increasing Demand and Partial Backlogging", Int. J Res. In Appl. Sci & Engng. Tech(IJRASET), 9(III), 2021, pp. 250-257.

[16] Neeraj Kumar, "Learning effect on an inventory model with two-level storage and partial backlogging under inflation", Int. J. Services and Operations Management, 16(1), 2013, pp. 105-122

[17] M K Sharma and K K Bansal, "Inventory Model for Non-Instantaneous Decaying Items with Learning Effect under Partial Backlogging and Inflation", Global Journal of Pure and Applied Mathematics. ISSN 0973-1768 13(6), 2017, pp. 1999-2008.

****